Logic 2

In maths and software

Propositional Logic

A proposition is any formal statement which is either true or false, but is not a matter of opinion.

A proposition can replace **p** in the following question:

Is it true that **p**?

Things that can be evaluated True or False

i. Set intersection is commutative, ie A n B is the same as B n A

ii. X Factor is the best programme on TV.

iii. Where are files hosted on StudySpace?

iv. Do not use your mobile phone in any of the buildings!

v. Cardiff is west of Swansea

vi. The number 7 is prime and the number 51 is not prime.

Equivalence Truth Table

Logical Equivalence Operator

p	q	p ⇔ q
T	T	T
T	F	F
F	T	F
F	F	T

p	q	(**not** p **or** q) **and** (**not** q **or** p)
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies and Contradictions

A proposition which is always True is a tautology.

For example:

A **or** (**not** A)

A $\Rightarrow$ A

not (A **and** B) $\Leftrightarrow$ **not** A **or not** B

NOTE: The truth tables for each of the above all have True throughout the final column

Tautologies and Contradictions

A proposition which is always False is a contradiction.

For example:

A **and** (**not** A)

not A **and not** B $\Leftrightarrow$ A **or** B

NOTE: The truth tables for each of the above all have False throughout the final column

Predicate Logic

There are many occasions when propositional logic is not expressive enough.

For example:

At least one student passes Maths.
All students pass Maths.
No students pass Maths.
At least one student doesn't pass Maths

We want a way to express the above where we can reuse the defined entities.

Predicate Logic

In propositional logic each statement would be assigned to a different propositional name.

In predicate logic one can define a single reusable predicate. — A Boolean Valued Function

That is:

M (s) : s passes Mathematics
or
M (n) : n passes Mathematics
or
M(x) : x passes Mathematics

} All the same

Predicate Logic

Now:

English	Predicate Logic
At least one student passes Maths.	$\exists s(M(s))$
All students pass Maths.	$\forall t(M(t))$
No students pass Maths.	$\forall n(not\ M(n))$
At least one student doesn't pass Maths.	$\exists x(not\ M(x))$

Predicate Logic

$\forall$ The universal quantifier -- all

$\exists$ The existential quantifier -- there exists

Some Translation: Predicate Logic to English

Assume the domain is students on this module, and we have the following predicates

E(x) : x is older than eighteen

B(y) : y has at least one brother

T(u,v) : u is taller than v

Translate the Following:

$$\exists p(B(p) \text{ and } E(p))$$

$$\forall m(E(m) \Rightarrow B(m))$$

$$\exists s \exists t(T(s,t))$$

$$\exists m \forall n(T(m,n))$$

$$\forall p \exists q(T(p,q))$$

Quantifier Stuff

$\forall x \forall y$

For any two....

$\exists m \exists n$

At least two...

$\forall s \exists t$

For every...there is a...

$\exists p \forall q$

There is a...for every...

Quantifiers and Negation

$not(\forall s\ P(s))$ $\quad$ $\exists r(not\ P(r))$

Not every...is...is equivalent to There is a...which is not

$not(\exists t\ W(t))$ $\quad$ $\forall d(not\ W(d))$

Not one...is... is equivalent to Every ... is not...

Variable Binding and Scope

Quantifiers bind variables by assigning them to a value from a domain.

Any unquantified variable in an expression is an unbound or free variable.

To determine the value of a predicate logic expression, all variables must be bound variables.

Variable Binding and Scope

$$\forall x(P(x)\ \ and\ \ Q(y))$$

In the above expression the variable x is bound and the variable y is free.

The extent of a variable binding is its scope. The scope of a variable binding is the predicate to its right or any expression contained with in parentheses if they immediately follow the Quantifier.

Variable Binding and Scope

$\forall x\, P(x)$ *and* Q(x)

The scope of x is P(x).
The second x is free.

$\forall x\, (P(x)$ *and* Q(x))

The scope of x is (P(x) and Q(x)).

$\forall x\ \ P(x)$ *and* $\exists$x Q(x)

The scope of the first x is P(x) and the scope of the second x is Q(x)

Evaluate the Following Expressions:

where the domain is the integers, LT (x, y) represents x < y, SQ (m, n) represents m is the square of n, and E (p) represents p is even.

$$\forall s \forall t\ \ LT(s, t)$$

$$\forall x \exists y\ SQ(y, x)$$

$$\exists r \forall s\ \ SQ(r, s)$$