Logic

In maths and software

Logic

Mathematics is built on logical reasoning.

Some types of logic:

propositional logic -- used to prove theorems

predicate logic -- used in conditional expressions (if-then-else) in any programming media (including spreadsheets)

temporal logic -- used to encode time-based information in artificial intelligence

Propositional Logic

A proposition is any formal statement which is either true or false, but is not a matter of opinion.

A proposition can replace **p** in the following question:

Is it true that **p**?

State Whether Each of the Following Is a Proposition or Not:

i. Set intersection is commutative, ie A n B is the same as B n A

ii. X Factor is the best programme on TV.

iii. Where are files hosted on StudySpace?

iv. Do not use your mobile phone in any of the buildings!

v. Cardiff is west of Swansea

vi. The number 7 is prime and the number 51 is not prime.

Logical Operators

Logical operators create new propositions from existing propositions.

and — Bamako is the capital of Mali **and** all relations are functions.
CONJUNCTION -- true when both are true

or — Bamako is the capital of Mali **or** all relations are functions.
DISJUNCTION -- true when at least one is true

not — Bamako is **not** the capital of Mali.
NEGATION -- true when original proposition is false

Translate the Following Expressions Into English

where p represents *you are under 25* and o represents *you are a regular user of Facebook*

1. p **and** o
2. o **and** p
3. p **or not** o
4. **not** p **and** o
5. **not** (p **and** o)
6. **not** (**not** p)

Reasoning About Propositions

Truth Tables
Semantic technique - used to calculate the value of an expression

Natural Deduction
Syntactic technique - inference rules that infer new facts from existing facts

Boolean Algebra
Syntactic technique - uses equational reasoning

Truth Tables

A truth table is a mechanism for determining all possible (truth) values of an expression. It is generated as follows:

i. the number of distinct propositional elements in the expression is determined.

ii. if n elements exist then 2^n rows are created.

iii. the columns of the table are headed by the component expressions. That is, the first n columns are headed by the propositional elements, and the following columns are headed by the subexpressions which depend only on these elements.

iv. the final column is headed by the original expression and its values are all the possible values of the original expression.

Truth Tables

For example:

The expression p and q has two propositional elements p, q.
We therefore require a truth table with 4 rows. That is:

HEADING	p	q	p **and** q
ROW 1			
ROW 2			
ROW 3			
ROW 4			

Truth Tables

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The expression p **and** q has two propositional elements p, q.
We therefore require a truth table with 4 rows. That is:

HEADING	p	q	p **and** q
ROW 1	T		
ROW 2	T		
ROW 3	F		
ROW 4	F		

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We require 4 rows because there are 4 different ways of assigning truth values to the propositional elements.

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ROW 1	T	T	T
ROW 2	T	F	F
ROW 3	F	T	F
ROW 4	F	F	F

The possible values of the expression

We require 4 rows because there are 4 different ways of assigning truth values to the propositional elements.

Truth Tables

One propositional element

p	**not** p
T	F
F	T

p	q	p **or** q
T	T	T
T	F	T
F	T	T
F	F	F

Two propositional elements

Truth Tables

and another example...

p	**not** p	**not** (**not** p)
T	F	T
F	T	F

Note that we have used three columns in the truth table. That is, one generates the truth values of subexpressions (such as **not** p) before generating the truth values of the whole expression.

More Examples

p	q	p **or** q	**not** (p **or** q)

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T		
T	F		
F	T		
F	F		

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	
T	F	T	
F	T	T	
F	F	F	

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	**not** p	**not** q	**not** p **and** **not** q

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	**not** p	**not** q	**not** p **and** **not** q
T	T			
T	F			
F	T			
F	F			

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	**not** p	**not** q	**not** p **and** **not** q
T	T	F		
T	F	F		
F	T	T		
F	F	T		

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	**not** p	**not** q	**not** p **and** **not** q
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

More Examples

p	q	p **or** q	**not** (p **or** q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	**not** p	**not** q	**not** p **and** **not** q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Conditional Propositions

If p then q

The above statement is logically correct if the truth of p implies the truth of q *or* if p is false.

NOTE: There is no cause-and-effect relationship between p and q

The Following Statements Are True

If 2 is a prime number then 6 is even.

If the moon orbits the earth then racecar is a palindrome.

If Barry is 4 feet tall then Budapest is the capital of Hungary.

If there is a finite number of primes then 3+2*5 = 25.

(Budapest *is* the capital of Hungary)

The Following Statements Are False

If 2 is a prime number then 7 is even.

If the moon orbits the earth then racecars is a palindrome.

If Barry is not 4 feet tall then Budapest is not the capital of Hungary.

If there is an infinite number of primes then 3+2*5 = 25.

Conditional Truth Table

Implication Operator

p	q	$p \Rightarrow q$	p	q	**not** p **or** q
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	F	F	T

Logical Equivalence

If p then q and if q then p

The above statement is logically correct if the truth of p implies the truth of q *and* if the truth of q implies the truth of p.

NOTE: There is no cause-and-effect relationship between p and q

The Following Statements Are True

If 2 is a prime number then 6 is even and vice versa.

The moon orbits the earth if and only if pip is a palindrome.

Barry is 4 feet tall is the same as saying Budapest is the capital of Turkey.

The Following Statements Are False

If 4 is a prime number then 6 is even and vice versa.

The moon orbits the earth if and only if Dan is a palindrome

Barry is not 4 feet tall is the same as saying Budapest is the capital of Turkey

Equivalence Truth Table

Logical Equivalence Operator

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	(**not** p **or** q) **and** (**not** q **or** p)
T	T	T
T	F	F
F	T	F
F	F	T

Tautologies and Contradictions

A proposition which is always True is a tautology.

For example:

A **or** (**not** A)

A $\Rightarrow$ A

not (A **and** B) $\Leftrightarrow$ **not** A **or not** B

NOTE: The truth tables for each of the above all have True throughout the final column

Tautologies and Contradictions

A proposition which is always False is a contradiction.

For example:

A **and** (**not** A)

not A **and not** B $\Leftrightarrow$ A **or** B

NOTE: The truth tables for each of the above all have False throughout the final column