Probability 2

Recap

The probability of any outcome is between 0 and 1

i.e. $0 \leq P(Oi) \leq 1$ for each i, and

The sum of the probabilities of all the outcomes equals 1

i.e. $P(O_1) + P(O_2) + \ldots + P(O_k) = 1$

As the probabilities add up to 1, sometimes its easier to think about the complimentary event

i.e. not P (E) = 1 - P (E)

The multiplication rule (AND rule) applies to independent events

i.e. P (A and B) = P (A) * P (B)

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Probability as a function

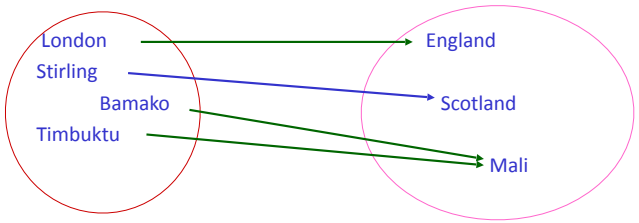
Probability can be described as a function

Functions: reminder

Function

A function can be represented as a mapping between two sets:

```
isATownIn
```

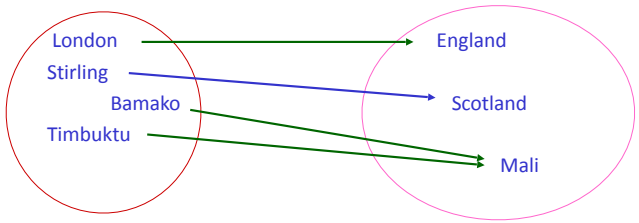


Functions: reminder

Function

Remember that many values from the domain can return only one value from the codomain

```
isATownIn
```

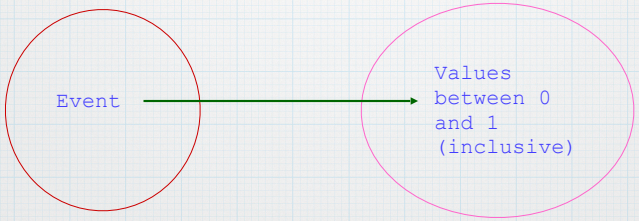


Probability as a function

Probability can be described as a function

To generate the probability of an event we apply a probability function to the event. Any probability function always returns a value between 0 and 1. That is:

```
probFunction :: Event -> Float01
```



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The multiplication rule (AND rule) applies to independent events

i.e. $P(A \text{ and } B) = P(A) * P(B)$

If there are only three outcomes $P(O_1)$, $P(O_2)$, $P(O_3)$ with

$P(O_1) = 0.2$
$P(O_2) = 0.5$

What is the probability of $P(O_3)$?

If $P(A) = 0.8$

What is the probability of $P(A^c)$?

What is $P(A \cup A^c)$?

Remember: union is like OR

Recap

The probability of any outcome is between 0 and 1

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The multiplication rule (AND rule) applies to independent events

i.e. $P(A \text{ and } B) = P(A) * P(B)$

AND exercise..

The following people are in a room: 5 of the 9 men are over 21, 6 of the 9 women are over 21. One person is chosen at random. The following events are defined:

A = { the person is over 21 }
B = { the person is 21 or under }

C = { the person is male } D = { the person is female }

Evaluate the following:

(a) P (A)
(b) P (B)
(c) P (C)
(d) P (D)

(e) P (Person is over 21 and male)
(f) $P(B \cap D)$
(g) $P(A \cap D)$
(h) $P(A \cap B)$

Remember: intersection is like AND

Sometimes we need to consider A or B

What is the probability that event A or event B will occur

Explain by example

Tossing a coin with rolling a 6 sided die

Coin \ Die	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

Tossing a coin with rolling a 6 sided die

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
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T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

Giving 7 / 12

Scenario

A large company has 200 members of management staff, of which 120 are male and 80 are female, working in one of three departments: HR, Accounts or Marketing.

Department	Male	Female	Total
HR	60	50	110
Accounts	24	16	40
Marketing	36	14	50
Total	120	80	200

Work out chances of ...

A male accountant or a female marketer winning the raffle

Department	Male	Female	Total
HR	60	50	110
Accounts	24	16	40
Marketing	36	14	50
Total	120	80	200

A male accountant or a female marketeer winning the raffle

Number who could win is 24 + 14 = 38

Giving 38 / 200 = 19 / 100 = 0.19

Department	Male	Female	Total
HR	60	50	110
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Marketing	36	14	50
Total	120	80	200

A male accountant or a female marketeer winning the raffle

Number who could win is 24 + 14 = 38

Giving 38 / 200 = 19 / 100 = 0.19

Note how P (MA or FM) = P (MA) + P (FM)

Note how P (MA or FM) = P (MA) + P (FM)

= 24 / 200 + 14 / 200

Giving 38 / 200 = 19 / 100 = 0.19

Watch out: only for this example - there is a catch!

The addition rule provides a way to work out the probability of A or B

P (A or B) = P (A) + P (B) - P (A and B)

Sometimes called the OR law

This bit is probably understandable P (A) + P (B)

.. but what about - P (A and B)

Explain by example

Using the OR rule: Tossing a coin with rolling a 6 sided die

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

probability of getting a head

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Coin	1	2	3	4	5	6
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Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

probability of getting a head 1/2

Using the OR rule: Tossing a coin with rolling a 6 sided die

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Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

probability of getting a 5

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What would be the probability of getting a head or a 5?

probability of getting a 5 1/6

Using the OR rule: Tossing a coin with rolling a 6 sided die

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

probability of getting a head 1/2 = 3/6

probability of getting a 5 1/6

Giving 3/6 + 1/6 = 4/6 = 2/3

Using the OR rule: Tossing a coin with rolling a 6 sided die

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

probability of getting a head 1/2 = 3/6

probability of getting a 5 1/6

Giving 3/6 + 1/6 = 4/6 = 2/3 <- Not Right!

Why?

We've double counted

Probability of getting a head = 6 / 12 = 1 / 2

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

We've double counted

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	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

We've double counted

Probability of getting a 5 = 2 / 12 = 1 / 6

Die

Coin		1	2	3	4	5	6
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Die

Coin

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Die

Coin

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Which is where we get a head and a 5

Back to the addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Back to the addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

The probability of A occurring

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The probability of A occurring

Back to the addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

The probability of both A AND B occurring

So

Die

Coin		1	2	3	4	5	6
	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

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Die

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probability of getting a head

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Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

P (A or B) = 6 / 12 + P (B) - P (A and B)

probability of getting a head

So

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

P (A or B) = 6 / 12 + 2 / 12 - P (A and B)

probability of getting a 5

So

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head or a 5?

P (A or B) = 6 / 12 + 2 / 12 - 1 / 12

probability of both

So

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

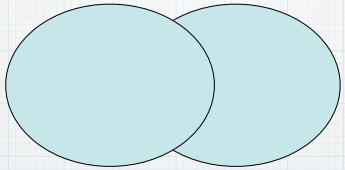
What would be the probability of getting a head or a 5?

P (A or B) = 6 / 12 + 2 / 12 - 1 / 12

P (A or B) = 7 / 12 <- Correct answer

Another way of looking at it ..

Event A Event B



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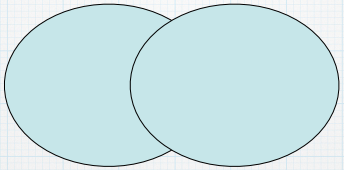

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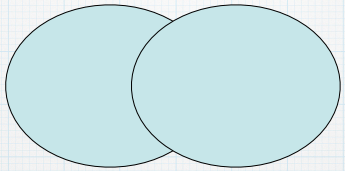
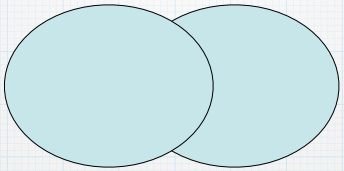


Calculating the probability of event A or event B

Calculating the probability of event A or event B

Event A + Event B



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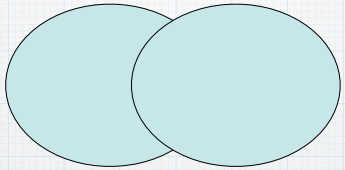

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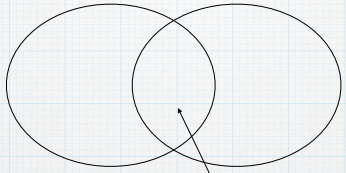
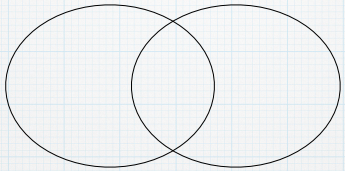


Double counts this area

Calculating the probability of event A or event B

Event A + Event B



P (A or B) = P (A) + P (B) - P (A and B)

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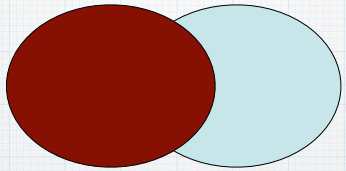
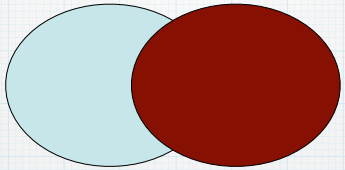


P (A or B) = P (A) + P (B) - P (A and B)

Calculating the probability of event A or event B

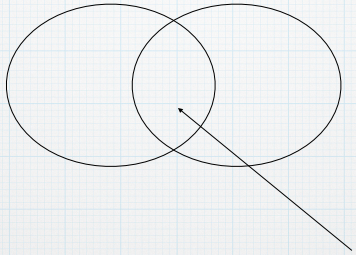
Event A + Event B



P (A or B) = P (A) + P (B) - P (A and B)

Calculating the probability of event A or event B

Event A + Event B



P (A or B) = P (A) + P (B) - P (A and B)

Another way of looking at it

Or typically means inclusive Or

Which include the AND case

Wearing a hat OR being over 6ft tall

You could be both (ie wearing a hat and be over 6ft tall)

So how does this work?

Department	Male	Female	Total
HR	60	50	110
Accounts	24	16	40
Marketing	36	14	50
Total	120	80	200

Work out chances of ...

A male accountant or a female marketer winning the raffle

There is no overlap so P (A and B) = 0

P (A or B) = P (A) + P (B) - P (A and B)

So how does this work?

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Total	120	80	200

Work out chances of ...

A male accountant or a female marketer winning the raffle

There is no overlap so P (A and B) = 0

P (A or B) = 24 / 200 + 14 / 200 ... etc

Exercises

In a large city, two newspapers are published, the Sun and the Post. The circulation departments report that 22% of the city's households have a subscription to the Sun and 35% subscribe to the Post. A survey reveals that 6% of all households subscribe to both newspapers. What proportion of the city's households subscribe to either newspaper?

That is, what is the probability of selecting a household at random that subscribes to the Sun or the Post or both?

If you select a card from a normal deck, what is the chance of getting a 2 or a red card?

Exercises

The following people are in a room: 5 of the 9 men are over 21, 6 of the 9 women are over 21. One person is chosen at random. The following events are defined:

A = { the person is over 21 }
B = { the person is 21 or under }

C = { the person is male } D = { the person is female }

Evaluate the following:

(a) P (A or D)
(b) P (C or D)
(c) P (A or B)

Remember this?

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of not getting a "tail and a 1 or 2"? <- Watch out! There's a trap here

Using the AND law

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of not getting a "tail and a 1 or 2"?

You may have done this ...

not (P (1 or a 2)) = 4/6
not (P (Tail)) = 1/2

Giving 4/6*1/2 = 4/12=1/3
Not correct - explanation soon ..

Rewrite

What would be the probability of not getting a "tail and a 1 or 2"?

probability of getting a "tail and a 1 or a 2"?

P (tail) and P (1 or a 2)

not (P (tail) and P (1 or a 2))

becomes

not (P (tail)) or not (P (1 or a 2))

But double counted so ..

not (P (tail)) or not (P (1 or a 2)) -
not (P (tail) and P (1 or a 2))

Rewrite

What would be the probability of not getting a "tail and a 1 or 2"?

not (P (tail)) + not (P (1 or a 2)) -
not (P (tail) and P (1 or a 2))

not (P (Tail)) = 1/2 + not (P (1 or a 2)) = 4/6
- not (P (tail) and P (1 or a 2)) = 4/12

Giving 1/2 + 4/6 - 4/12 = 6/12 + 8/12 - 4/12
= 10/12 = 5 / 6

Using the AND law

Four coins are tossed - what is the chance of getting 4 tails?

Two cards are drawn from a pack and then replaced -
what is the chance of getting an ace followed by a king?

The word replaced is important here - why?

John cycles to work on average 3 days a week. Bill cycles twice a week. What is the chance that they both cycle?