Probability 3

Conditional Probability

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Or

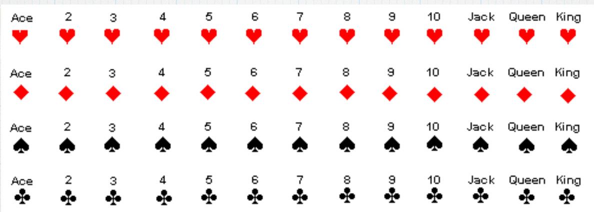
$$P(A \text{ and } B) = P(A)\, P(B \mid A)$$

Where $P(B \mid A)$ means the probability of B occurring given that A has occurred

So where does this come from?

Need to think about independent and dependent events

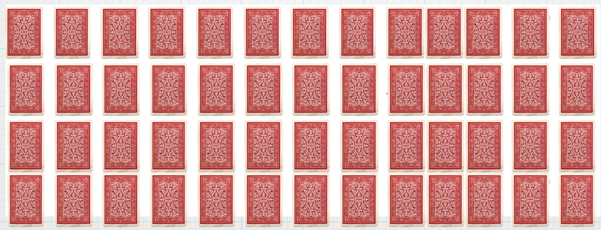
Consider taking two cards from a pack of cards what is the chance of getting two kings?



Need to think about independent and dependent events

Consider taking two cards from a pack of cards what is the chance of getting two kings?

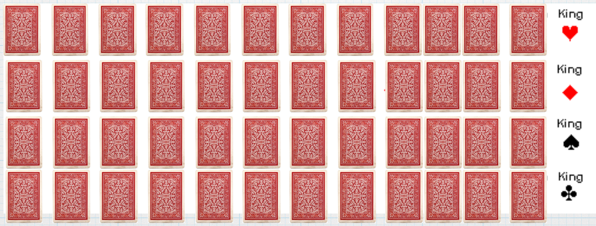
Take one card ... what's the chance of getting a king?



One card - four kings in the pack

Hence

4 / 52 chance of getting a king



Now we have a choice

Put the card back

Creates an independent event

Don't put the card back

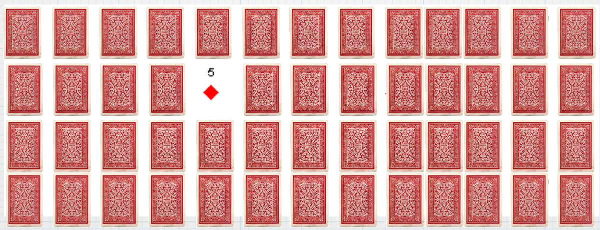
Creates a dependent event

Put the card back

Creates an independent event

The first card selected has no effect on the second card selected

The pack has been reset



So .. Chance of getting two kings if first card is replaced

P (Getting a king) = 4 /52

P (Getting second king) = 4 /52

So probability of getting a king and a king

Addition rule i.e. P (A and B) = P (A) * P (B)

P (Getting two kings) = 4 / 52 * 4 / 52

= 1 / 169

But what about the other choice

Put the card back

Creates an **independent** event

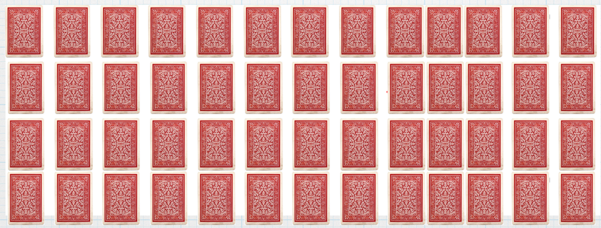
Don't put the card back

Creates a **dependent** event

Don't put the card back

Creates an **dependent** event

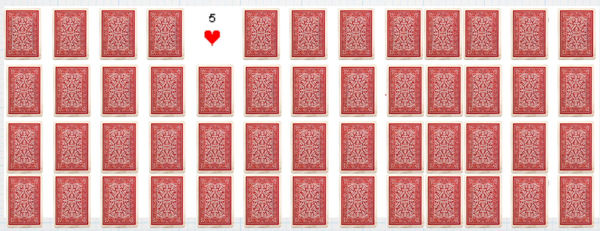
The first card selected changes which second card can be selected



Don't put the card back

Creates an **dependent** event

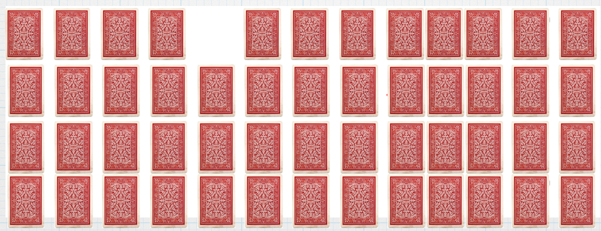
Pick a **5**



Don't put the card back

Creates an **dependent** event

Now there are **51** cards with **4** kings left



Don't put the card back

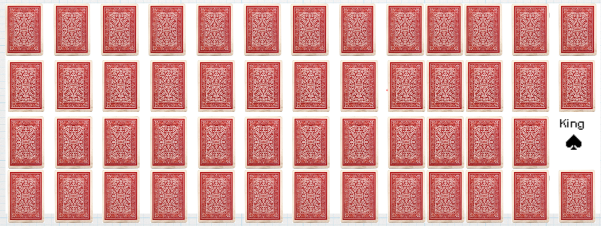
Creates an dependent event

Or ... pick a King

Don't put the card back

Creates an dependent event

Or ... pick a King



Don't put the card back

Creates an dependent event

Now there are 51 cards
with 3 kings left

So ..

Chance of getting two kings
if first card is not replaced

P (Getting first king) = 4 / 52

P (Getting second king given first king selected) = 3 / 51

So probability of getting a king and a second king

Addition rule

P (A and B) = 4 / 52 * 3 / 51

= 12 / 2756

= 3 / 689

So ..

Chance of getting two kings
if first card is **not** replaced

P (A) = 4 / 52

P (Getting second king given first king selected) = 3 / 51

So probability of getting a king **and** a second king

Addition rule

P (A and B) = 4 / 52 * 3 / 51

= 12 / 2756

= 3 / 689

So ..

Chance of getting two kings
if first card is **not** replaced

P (A) = 4 / 52

P (B | A) = 3 / 51

So probability of getting a king **and** a second king

Addition rule

P (A and B) = 4 / 52 * 3 / 51

= 12 / 2756

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So ..

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Conditional Probability

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

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Where $P(B \mid A)$ means the probability of B occurring given that A has occurred

Try these

A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

In the UK, 88% of all households have a television. 51% of all households have a television and a DVD. What is the probability that a household has a DVD given that it has a television?

Activity

There were 1200 police officers who could have been promoted over the last two year. 288 Men and 36 Women were promoted, 672 Men and 204 Women failed to get promoted.

Draw a joint probability table for this information

joint probability table

	Men	Women	Totals
Promoted			
Not promoted			
Totals			

joint probability table

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that a Man got promoted ?

What is the probability that a Woman didn't get promoted ?

What is the probability of promotion overall ?

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that an officer got promoted given that the officer is a man?

Two different ways to work it out

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that an officer got promoted given that the officer is a man?

By formula $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that an officer got promoted given that the officer is a man?

By formula $P(B \mid A) = \dfrac{P(\text{Being male and Promoted})}{P(\text{male})}$

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that an officer got promoted given that the officer is a man?

By formula $P(B \mid A) = \dfrac{288 / 1200}{960 / 1200}$

$= \dfrac{288}{960}$

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that an officer got promoted given that the officer is a man?

Second way: read it directly from the grid

	Men	Women	Totals
Promoted	288	36	324
Not promoted	672	204	876
Totals	960	240	1200

What is the probability that an officer got promoted given that the officer is a man?

By formula $P(B \mid A) = \dfrac{288 / 1200}{960 / 1200}$

$= \dfrac{288}{960}$

Note how both methods give the same result

independent events

Two events A and B are independent, if

$$P(A \mid B) = P(A)$$

Or

$$P(B \mid A) = P(B)$$

Otherwise the events are dependent

Don't confuse mutual exclusion and independent events

Mutually exclusive events are dependent

If an event is mutually exclusive - then if it occurs the probability of the other occurring is reduced to zero - hence they are dependent