Probability

For every problem with an uncertain result there is a set of possible outcomes. This is known as the sample space

Example sample space

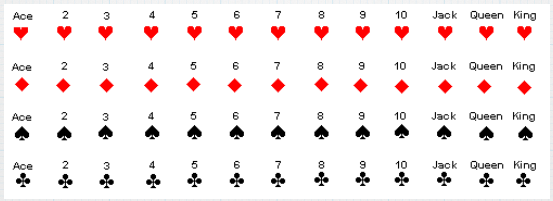
Outcomes for a six sided die

1 2 3 4 5 6

For every problem with an uncertain result there is a set of possible outcomes. This is known as the sample space

Example sample space

Outcomes for pack of cards



The best way to represent the sample space is to use the set notation

Example sample space

Outcomes for a six sided die

1 2 3 4 5 6

Sample space for a six sided die $S = \{ 1, 2, 3, 4, 5, 6 \}$

Write down the sample spaces for

- Coin toss outcomes
- BIA student ages
- Favourite soap opera
- Result grades for coursework
- Letters in the word "mathematics"

The sample space must be exhaustive and mutually exclusive

exhaustive must show all the outcomes

Sample space for a six sided die $S = \{ 1, 2, 3, 5, 6 \}$ ✘

$S = \{ 1, 2, 3, 5, 4, 6 \}$ ✔

mutually exclusive no overlap between outcomes

Sample space for numbers less than 6

$S = \{$ number less than 6 or even number$\}$ ✘

$S = \{ 1, 2, 3, 5, 4, 6 \}$ ✔

An event of an experiment or action is a subset from the sample space

Sample space for a six sided die $S = \{ 1, 2, 3, 4, 5, 6 \}$

events

Getting an odd number	$E = \{ 1, 3, 5 \}$
Getting an even number	$E = \{ 2, 4, 6 \}$
Getting a 2	$E = \{ 2 \}$
Getting a number less than 4	$E = \{ 1, 3, 2 \}$

Write down the events for

events	sample space
Getting an even age	BIA student ages
Getting a BBC soap opera	Favourite soap opera
Getting a grade better than a B	Result grades for coursework
Getting a vowel	Letters in the word "mathematics"

There are three approaches to generating probabilities:

A priori classical approach - use previous knowledge about the problem.

THE PROBABILITY OF AN INTEGER BEING EVEN

An empirical classical approach. This is based on observations of the problem.

THE PROBABILITY OF A CAR IN THE CAR PARK BEING BLUE

A subjective approach.

THE PROBABILITY OF A LECTURE BEING INTERESTING

Given a sample space $S = \{O_1, O_2, ..., O_k\}$, the probabilities assigned to the outcome must satisfy these requirements:

The probability of any outcome is between 0 and 1

i.e. $0 \leq P(O_i) \leq 1$ for each i, and

The sum of the probabilities of all the outcomes equals 1

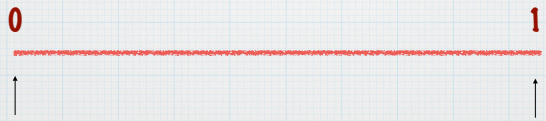
i.e. $P(O_1) + P(O_2) + ... + P(O_k) = 1$

The probability of any outcome is between 0 and 1

i.e. $0 \leq P(Oi) \leq 1$ for each i, and

Every probability will be a number

between 0 and 1 (inclusive)



Will never occur

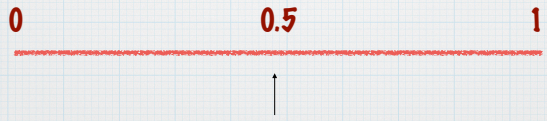
Will always occur

The probability of any outcome is between 0 and 1

i.e. $0 \leq P(Oi) \leq 1$ for each i, and

Every probability will be a number

between 0 and 1 (inclusive)



Is equally likely to occur or to not occur

We sometimes use percentages ..

It's 20% likely to occur

It's 55% likely to occur

17% likely

Never happen - 0% likely

Will always happen - 100% likely

.. or we can use decimals

It's 20% likely to occur	0.2
It's 55% likely to occur	0.55
17% likely	0.17
Never happen - 0% likely	0
Will always happen - 100% likely	1

.. or we can use fractions

It's 20% likely to occur	1/5
It's 55% likely to occur	11/20
17% likely	17/100
Never happen - 0% likely	0
Will always happen - 100% likely	1

The sum of the probabilities of all the outcomes equals 1

i.e. $P(O_1) + P(O_2) + \ldots + P(O_k) = 1$

Sample space for a six sided die $S = \{ 1, 2, 3, 4, 5, 6 \}$

1	2	3	4	5	6
↑	↑	↑	↑	↑	↑
1/6	1/6	1/6	1/6	1/6	1/6

The sum of the probabilities of all the outcomes equals 1

i.e. $P(O_1) + P(O_2) + \ldots + P(O_k) = 1$

Sample space for a six sided die $S = \{ 1, 2, 3, 4, 5, 6 \}$

1 2 3 4 5 6

1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6

= 6/6 = 1

When the outcomes are equally likely, the probability of an event is given by

$$\frac{\text{number of wanted outcomes in E}}{\text{number of possible outcomes in S}}$$

Often written as $P(E)$

Work out these probabilities

events	sample space
Getting an even number	Six sided die
Getting an number less than 3	Six sided die
Getting a vowel	Letters in the word "mathematics"

Work out these probabilities

events	sample space
Getting an even age	BIA student ages

As the probabilities add up to 1, sometimes its easier to think about the complimentary event

not P (E) = 1 - P (E)

There's a 3/6 chance of getting an even number on a 6 sided die

There's a 1 - 3/6 chance of getting an odd number

As the probabilities add up to 1, sometimes its easier to think about the complimentary event

not P (E) = 1 - P (E)

There's a 1/7 chance of catching the bus - so not catching the bus?

There's a 60% chance of rain - so not raining?

There's a 0.34 chance of not winning the race - so winning?

What's the chance of not picking an Ace from a pack of cards ?

Scenario A large company has 200 members of management staff, of which 120 are male and 80 are female, working in one of three departments: HR, Accounts or Marketing.

Department	Male	Female	Total
HR	60	50	110
Accounts	24	16	40
Marketing	36	14	50
Total	120	80	200

Work out chances of ...

A woman winning the raffle

A man wins the raffle

A man from the Accounts department wins the raffle

Bring on the personal trainers

Probability that computerisation will lead to job losses within the next two decades, 2013 (1=certain)

Job	Probability
Recreational therapists	0.003
Dentists	0.004
Athletic trainers	0.007
Clergy	0.008
Chemical engineers	0.02
Editors	0.06
Firefighters	0.17
Actors	0.37
Health technologists	0.40
Economists	0.43
Commercial pilots	0.55
Machinists	0.65
Word processors and typists	0.81
Real estate sales agents	0.86
Technical writers	0.89
Retail salespersons	0.92
Accountants and auditors	0.94
Telemarketers	0.99

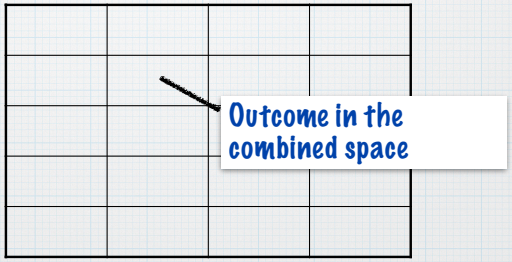
Source: "The Future of Employment: How Susceptible are Jobs to Computerisation?" by C.Frey and M.Osborne (2013)

When two or more events can occur together (or after each other) - they give a **compound** or **combined** event

Can often be represented as a table

list of outcomes in one sample space

list of outcomes in the other sample space



Consider tossing a coin and rolling a 6 sided die

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

So sample space S would be ...

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head and a 5?

What would be the probability of getting a tail and a 1 or 2?

What would be the probability of not getting a "tail and a 1 or 2"?

If two (or more) events are **independent** if the outcome of one event has **no** effect on the other

the probability of wearing red and being on the bus?

the probability of being 18 and being left handed?

the probability of getting a head and a 5?

The multiplication rule applies to independent events

$$P(A \text{ and } B) = P(A) * P(B)$$

Sometimes called the AND rule

It is VITAL that they are independent events

Using the AND rule

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of getting a head and a 5?

What would be the probability of getting a tail and a 1 or 2?

What would be the probability of not getting a "tail and a 1 or 2"? <- Watch out! There's a trap here

Using the AND rule

Die

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

What would be the probability of not getting a "tail and a 1 or 2"?

You may have done this ...

not (P (1 or a 2)) = 4/6

not (P (Tail)) = 1/2

Giving 4/6*1/2 = 4/12=1/3

Not correct - explanation soon ..

Using the AND rule

Four coins are tossed - what is the chance of getting 4 tails?

Two cards are drawn from a pack and then replaced - what is the chance of getting an ace followed by a king?

The word replaced is important here - why?

John cycles to work on average 3 days a week. Bill cycles twice a week. What is the chance that they both cycle?